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# Self-Similar Cosmological Model: Technical Details, Predictions, Unresolved Issues, and Implications

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This second paper of a two-part review of the self-similar cosmological model includes a derivation of the self-similar scale transformation equations, discussions of various details of the model, and a survey of the definitive predictions by which the model can be unambiguously tested in the near future. Unresolved issues and problems are identified and discussed. The review concludes with an examination of the theoretical implications of discrete cosmological self-similarity.

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## 1. INTRODUCTION

The first paper (Oldershaw, 1989*a*; hereafter Paper I) of this two-part review of the self-similar cosmological model (SSCM) introduced the general concepts of the model, gave several motivations for its consideration, and presented 20 retrodictive tests based on the model's discrete scale transformation equations. In this second half of the review the SSCM is explored in greater detail. The derivation of the discrete self-similar scale transformation equations is recapitulated and various technical details of the model are investigated. Nine definitive predictions that can serve as rigorous and unambiguous tests for verifying/falsifying the model are described. Also included are wide-ranging discussions of unresolved issues, problematic conflicts between the model and present knowledge, and theoretical implications of discrete cosmological self-similarity.

In Paper I the degree of self-similarity, defined as the degree to which analogous phenomena on different cosmological scales are quantitatively correlated by self-similar scaling relations, was not fully specified, although the 20 successful empirical retrodictions suggested that the proposed scale

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transformation equations are good to a factor of 2 or better. In this paper the strong principle of self-similarity (Oldershaw, 1986*a*) is *tentatively* adopted. This principle proposes that all cosmological scales have completely equivalent systematic organizations and that the physical properties of analogue systems from different cosmological scales are completely equivalent except in relative mass-space-time scale; the proposed scale transformation equations are regarded as good approximations to the actual transforms, which are discrete, linear, and exact. Such a strict form of the SSCM may be an overidealization, but tentatively adopting the strong principle of self-similarity greatly simplifies the exposition of the model. Moreover, the conceptual simplicity, the high degree of symmetry, and the remarkable capacity for unification that characterize the exact self-similarity version of the SSCM make it the most logical initial interpretation of the evidence for discrete cosmological self-similarity. In the long run, increasing empirical knowledge of the three observable cosmological scales will gradually reveal whether exact self-similarity is an overidealization. In this paper it is employed as a working first approximation to the actual degree of cosmological self-similarity.

## 2. TECHNICAL DETAILS

### 2.1. Derivation of the Self-Similar Scale Transformation Equations

The general concept of a discrete self-similar cosmos occurred to the author on 21 December 1976, but it took another 8 years to arrive at proper scale transformation equations. This was largely due to an inappropriate set of analogies based on qualitative morphological and kinematic considerations (Oldershaw, 1986*a*). These considerations led to an assumption that a Rydberg atom, the Solar System, and the Local Group of galaxies were appropriate analogues from the three observable cosmological scales. If the SSCM is a valid model, then it is now without question that a Rydberg atom and the Solar System are correct analogues, but the Local Group is an inappropriate galactic scale analogue with which to complete the three-member set. This erroneous analogy implied a highly asymmetric cosmological hierarchy with strongly nonlinear scaling, a plausible cosmological model, but never one that the author felt satisfied with. Fortunately, in early 1985 the author finally recognized that inappropriate stellar–galactic analogies were the source of the problem and that a symmetric cosmological hierarchy with linear scaling was in good accord with basic *empirical* knowledge of atomic, stellar, and galactic scale systems. Since the SSCM is so dependent upon the scale transformation equations derived in 1985, it is appropriate to carefully reiterate their derivation.

### 2.1.1. *Form of the Equations*

Because of the considerable evidence (Paper I) indicating that nature's hierarchy is highly stratified, it was anticipated that the self-similar scale transforms should be discrete rather than continuous. Given the simplest assumptions—that the equations are linear and that time should be treated as being closely analogous to spatial dimensions in a 4-dimensional space-time—one arrives at the following form for the discrete self-similar scaling equations:

$$R_N = \Lambda R_{N-1} \tag{1}$$

$$T_N = \Lambda T_{N-1} \tag{2}$$

$$M_N = X M_{N-1} \tag{3}$$

where $R$, $T$, and $M$ are length, temporal period, and mass values characterizing analogue systems from neighboring scales $N$ and $N-1$, and where $\Lambda$ and $X$ are dimensionless scaling constants. A generic relationship (Mandelbrot, 1982) for self-similar systems is

$$n = (R_N / R_{N-1})^D \tag{4}$$

where $n$ is the number of $N-1$ scale systems that constitute an analogous system on scale $N$, the $R$ values are radii of the analogue systems, and $D$ is a dimensionless self-similarity constant that is often referred to as the "fractal dimension." Previous discussions (Oldershaw, 1986*b*) of the SSCM have used the crude approximation that $n = M_N/M_{N-1}$, and since $R_N/R_{N-1} = \Lambda$, one arrives at the following relation for the scaling of analogue masses:

$$M_N = \Lambda^D M_{N-1} \tag{5}$$

However, as will be discussed in Section 4.7, it is not certain that $\Lambda^D$ is an appropriate expression for the constant $X$ in (3). This doubt arises from the question of whether (4) is applicable to a self-similar hierarchy that is dominated by completely collapsed objects (i.e., black holes) wherein systems of scale $N$ have only a peripheral substructure of $N-1$ scale analogues that account for only a tiny fraction of the total mass of the scale-$N$ system. Equation (5) is the mass transform that is used in this paper, but it should be kept in mind that one might have to retreat to the more general form embodied in equation (3).

### 2.1.2. *Derivation of the Scaling Constants*

In order to derive an empirical value for $\Lambda$, one needs to evaluate $R_N/R_{N-1}$ for analogue systems that are in equivalent energy states. Since reasonably accurate data are available for atomic scale systems, the difficulty lies primarily in finding comparably accurate radius data for a stellar scale analogue. The Solar System is virtually the only available choice here, and it is immediately obvious that if the Solar System is rigorously analogous to an atomic scale system, then the latter *must* (Oldershaw, 1982, 1986*b*) be a Rydberg atom with very high principal and orbital quantum numbers. Atoms in such states obey two general relations that give classical approximations to the orbital radius and velocity of the "planetary" electron:

$$r \approx n^2 a_0 \tag{6}$$

and

$$v \approx v_0 / n \tag{7}$$

where $r$ is the orbital radius, $n$ is the principal quantum number, $a_0$ is the Bohr radius, $v$ is the orbital velocity, and $v_0$ is the classical velocity of the ground-state electron. Of course, quantum mechanics is currently the most rigorous theory of atomic physics, but in keeping with the correspondence principle, the classical and quantum models converge as $n$ becomes large. For very high values of $n$ and $l$ (the orbital quantum number), the predictions based on the classical model *and the qualitative aspects* of the classical model are quite accurate. The value of $r$ in (6) approximates the radius at which the radial wavefunction, usually interpreted as the probability for finding the electron, is at a maximum. As will be discussed in Section 4.3, the SSCM strongly endorses an alternative interpretation (Barut, 1988) of quantum mechanics, first proposed by Schrödinger, that views the wavefunction as a distribution of mass (or charge) rather than probability. The *bound* electron is seen as having its mass (or charge) distributed throughout the atom. Therefore, the SSCM interprets the value of $r$ in (6) as the radius at which the mass distribution has its peak. If the Solar System is a physically meaningful analogue to a Rydberg atom, then one may solve (Oldershaw, 1986*b*) equations (6) and (7) for a stellar scale "Bohr radius" ($A_0$) by using the orbital radius ($r_J$) and velocity ($v_J$) of Jupiter, which overwhelmingly dominates the distribution of planetary mass, as the appropriate stellar scale values for $r$ and $v$ in the Solar System. Since velocities are regarded as scale invariant in the SSCM, (7) can be solved for the Solar System value of $n$:

$$n \approx v_0 / v_J \approx 168 \tag{8}$$

This value for $n$ can then be used along with $r_J$ to solve (6) for $A_0$:

$$A_0 \approx r_J / n^2 \approx 2.8 \times 10^9 \text{ cm} \tag{9}$$

Finally,

$$A_0 / a_0 \equiv \Lambda \approx 5.2 \times 10^{17} \tag{10}$$

In order to derive the mass scaling constant $X$ of equation (3), or $\Lambda^D$ of equation (5), one may not use the Solar System, because, although it is identified as an atom in a Rydberg state, it is not possible at this point to determine the specific atomic element to which the Solar System is analogous. The most direct alternative strategy is to determine the stellar scale analogue to the hydrogen atom and take a ratio of the masses of these two analogues. Given that the lower limit radius for the stellar scale H analogue should be about $3A_0$ [since the corresponding radius (Oldershaw, 1986*b*) for the H atom is of the order of $3a_0$] and that its abundance should be about 90% of all stars (since H comprises about 90% of all atoms), one can unambiguously identify $M$ dwarf stars as the correct analogue. The estimation of stellar masses is very difficult and imprecise, but the peak of the distribution of masses for $M$ dwarf stars appears to be at about $0.15M_\odot$ (Oldershaw, 1986*b*). This approximate mass for the stellar scale H analogue can be independently checked by the following method. In Paper I it was noted that the central stars of planetary nebulae are primarily $\mathrm{He}^+$ analogues, and since this class of stars has a remarkably discrete mass peak at about $0.58M_\odot$, the approximate mass of the stellar scale H analogue should be four times less than this value, or about $0.145M_\odot$. Therefore,

$$X \approx 0.145M_\odot/1.67\times10^{-24}\,g \approx 1.73\times10^{56} \tag{11}$$

and if $X = \Lambda^D$, then $D \approx 3.174$.

As mentioned in Paper I, critics of the SSCM have argued that the successful retrodictions achieved by the discrete scaling equations of the SSCM have been the result of fortuitous coincidences or have been due to manipulative choices of analogues, scaling constants, and tests. If one carefully and *objectively* studies the derivation of the scaling equations and the 20 successful retrodictions based upon them, then one will arrive at the following conservative conclusions. Each step of the derivation of $\Lambda$ and $D$ and the initial choices of analogues are constrained and do not permit the degree of manipulation invoked by critics. Once the scaling equations are found, the identification of analogues is even more highly constrained. Moreover, the majority of the successful retrodictions were achieved *after* the scaling equations had been submitted for publication. Finally, coincidence is an entirely unscientific explanation for 20 successful retrodictions of *diverse* and *fundamental* physical parameters for systems on vastly different size scales (Paper I). Therefore, one is inevitably led to the conclusion that discrete cosmological self-similarity is a real, but previously unappreciated, attribute of nature.

## 2.2. Defining Cosmological Scales and Identifying Analogues

In Paper I the concept of cosmological scales was introduced on an informal basis as a way of interpreting the empirical fact that nature's nested hierarchy is highly stratified. A testament to this inherent stratification is that anyone with a broad knowledge of the natural world intuitively has a general understanding of what one means by atomic, stellar, and galactic scales, whereas this would be much less the case if nature's hierarchy were poorly stratified. But Paper I did not include a rigorous discussion of the idea of cosmological scales, and so this important subject will now be explored more fully. A previous paper (Oldershaw, 1985) that presented detailed discussions of nature's nested hierarchical organization in terms of abstract set theory and actual physical objects will serve as a general reference to the following review of the basic features of cosmological scales in the SSCM.

As has been noted above, the *observable portion* of nature's nested hierarchy can be naturally decomposed into atomic, stellar, and galactic scale systems. The atomic scale is selected for study as the "archetypal" cosmological scale because our knowledge of its organization and the physical properties of its constituent systems is much more comprehensive than our knowledge of the stellar scale, which in turn dwarfs our knowledge of the galactic scale.

Consider a formal ordering, in terms of rest mass, of all distinct and homogeneous classes of *stable* (or at least quasistable), *massive* systems. Massless systems will not be treated here except to note that the SSCM proposes that whatever their physical properties are, all such systems will have self-similar analogues on all scales. The class of least massive systems that constitute the lowermost *level* of the SSCM's atomic scale is the class of electrons. At a mass of about 1836 times that of the electron, the class of protons constitutes the next highest level, and slightly above this is a level for the quasistable neutron. The particles from these three "elementary" levels of the atomic scale can be bound in various permitted (stable) combinations so as to yield a discrete hierarchy of levels corresponding to the atomic elements. Each of these latter levels has a fine structure due to various ionization states for each atom. The SSCM usually defines the three elementary levels and the atomic levels as the atomic *scale* of the cosmological hierarchy, although this definition is somewhat flexible (Oldershaw, 1985), e.g., unstable systems could be included or the upper cutoff could be chosen differently. Immediately above the defined atomic scale is a very large collection of ordered levels corresponding to classes of molecules, and this series of levels is referred to as the molecular levels. Between the upper molecular levels and the first level of the stellar scale there is an incredibly large, populous, and complex band of levels representing everything from the lightest pieces of stable conglomerate matter to objects approaching $7\times10^{-5}M_{\odot}$. In this band of macroscopic levels the sets are fuzzy at best; classes of systems here do not have discrete masses and there is considerable overlapping of mass ranges for different levels. The molecular levels and the large band of macroscopic levels are collectively defined as the atomic–stellar *interscale region* of the cosmological hierarchy. The even larger combined set of the atomic scale and the interscale levels is referred to as the $N = -1$ *subhierarchy* of the cosmological hierarchy. $N$ values designate particular subhierarchies and the subhierarchy that includes the stellar scale has been arbitrarily given the value $N = 0$. The SSCM asserts that above the atomic–stellar interscale region the entire hierarchical pattern just described repeats itself. There is a stellar scale of discrete systems consisting of three levels of "elementary" systems with masses equal to $\Lambda^D$ times the $e^-$, $p^+$, and neutron masses and a large number of levels populated by stellar analogues to atoms. This is followed by a vast number of levels representing the molecular and macroscopic systems of the stellar–galactic interscale region. The organizational pattern of the $N = -1$ subhierarchy is hypothesized to repeat without end throughout the unbounded cosmological hierarchy. Since this discussion has involved very radical conjectures, it is appropriate to emphasize that the reality of a stellar scale can be tested in the following straightforward manner, as will be discussed in more detail in Section 3.1. If there is meaning to the idea of a stellar scale that is formally equivalent to the atomic scale, then about 90% of the enigmatic dark matter *must* (Oldershaw, 1986*c*) be in the form of a huge number of black holes that all have the same mass of about $0.145 M_{\odot}$. If this is not the case, then the SSCM will have been unambiguously falsified. If, however, this unique and radical prediction is verified, then the SSCM will have solved what is perhaps the most fundamental scientific problem of our era (nature of the dark matter), and one on which all previous theories and paradigms will have foundered.

Having defined the concepts of cosmological levels, scales, interscale regions, and subhierarchies, it will be useful to summarize briefly the previously identified (see Paper I) analogue pairs from different scales. Again, atomic scale systems are used as reference scale systems for the reasons cited above. *M* dwarf stars, *K* dwarf stars, and the majority of white dwarf stars have been proposed as stellar scale analogues of hydrogen atoms, helium atoms, and $He^+$ ions, respectively. Mid-to-upper main sequence, giant, and supergiant stars have been identified as stellar scale counterparts of Rydberg atoms and ions in low to very high energy states. Variable stars are analogous to Rydberg atoms that are actively undergoing energy state transitions, and *RR* Lyrae stars have been specifically identified with neutral He atoms undergoing transitions between states with $n$ equal to 7, 8, and 9. Neutron stars have been proposed as analogues to atomic nuclei, primarily with masses larger than helium nuclei. The Solar System appears to be analogous to a lithium atom with $n_1 \approx 1$, $n_2 = 5$, and $n_3 \approx 168$ (with $l_3 \approx n_3$). The radii and peculiar velocities of galaxies unambiguously identify them as galactic scale analogues to atomic nuclei under conditions of very high temperature and density. Consistent with this analogy is the observed clustering of galaxies into sheets and filaments interspersed with very large scale "voids," which is similar to the clustering expected to occur in very high temperature atomic scale plasmas. Also, the shapes of galaxies, to the extent that they can be inferred without a full knowledge of the distribution of galactic dark matter, are of the same types as those found for atomic scale nuclei, and both atomic nuclei and elliptical galaxies can have "flattened" shapes that are not completely explicable in terms of rotational forces (Oldershaw, 1985). Finally, as will be discussed in more detail in Section 4.3, globular clusters have radii and a cosmological abundance that are consistent with expectations for galactic scale electron analogues. In principle, given equations (1)-(3), one can choose any stellar or galactic object that is physically well defined (i.e., reasonably accurate mass, radius, spin period, etc.) and determine its atomic scale counterpart. The one caveat is that one *must* remember to compare systems that are in equivalent energy states. A case in point is the fact that the morphological and kinematic properties of hydrogen in its ground state are radically different from those of hydrogen in a Rydberg state with $n > 100$ and $l \approx n$.

## 2.3. Relativity of Cosmological Scale

Immediately after the scale transformation equations were developed in 1985, a curious result was noted (Oldershaw, 1986*b*). The predicted radius for the stellar scale proton analogue was found to be approximately equal to the Schwarzschild radius $R_{\mathrm{Sch}}$ for an object of $0.145 M_{\odot}$, its predicted mass. However, the conventional $R_{\mathrm{Sch}}$ for the proton was smaller than the proton radius by a huge factor, and the conventional $R_{\mathrm{Sch}}$ for the galactic scale proton analogue was larger than the typical galactic radius by a similarly huge factor. This problem was resolved by recognizing that dimensional constants, in this case the Newtonian gravitational constant $G$, must be scaled according to the same rules as lengths, times, and masses. In a subsequent paper (Oldershaw, 1986*a*) it was noted that this type of scaling strongly suggests that the scale transformation equations relate equivalent sets of units on different cosmological scales. Therefore, in any comparison of physical properties for analogues on different cosmological scales, such as their Schwarzschild radii, *all* dimensional quantities must be scaled. One has the option of saying that the atomic scale value of $G$ is equal to $(6.67 \times 10^{-8}\,\mathrm{cm}^3/\mathrm{g\,sec}^2)\ (\Lambda^D \Lambda^2/\Lambda^3) = 1.85 \times 10^{31}\,\mathrm{cm}^3/\mathrm{g\,sec}^2$ using conventional cgs units, or that it is equal to the familiar value of $6.67 \times 10^{-8}\,\langle\mathrm{cm}\rangle/\langle\mathrm{g}\rangle\langle\mathrm{sec}\rangle^2$, but that the atomic scale ⟨cgs⟩ units have been scaled according to equations (1)-(3). [Parenthetically, it should be mentioned that the signs of the exponents in equation (6) of the paper just cited have been inadvertently switched, but that the right-hand side of the equation is correct.]

Therefore, the SSCM views cosmological scales as being formally equivalent, and if the units of different cosmological scales are determined in an equivalent manner, then one arrives at equivalent sets of units that differ only in relative scale. While conventional scaling would still apply *within* a single cosmological scale, only relative scaling would be possible between different cosmological scales. For example, if one only considers the atomic scale and defines a unit (say the angstrom, $10^{-8}$ cm), then there is no confusion about what is meant by a length measured in angstroms. But if one includes all other cosmological scales in the discussion, then there are an infinite number of "angstrom" units, i.e., an atomic scale "angstrom," a stellar scale "angstrom," a galactic scale "angstrom," etc., each with an equal claim to validity. Their relative magnitudes, measured with respect to an arbitrarily chosen reference scale "angstrom," would scale according to equations (1)–(3). Science has always explicitly or implicitly assumed that the magnitudes of defined units and the conventional concept of scale have unique and absolute meaning, but if the SSCM is correct, then this assumption will have to yield to the hypothesis that the magnitudes of units and the concept of scale have only relative meaning in the cosmological context.

## 2.4. An Unbounded Cosmological Hierarchy

In Paper I it was suggested that nature's nested hierarchy *might* be unbounded in scale (and therefore in space-time as well), i.e., that there is no such thing as a smallest or a largest object in nature, but rather that there is an endless succession of ever-smaller and ever-larger objects. If the strong principle of self-similarity is correct, as is tentatively hypothesized in this paper, then the cosmological hierarchy *must* be completely unbounded, since only in the case of an infinite hierarchy can a given system and its lower scale analogues be totally equivalent except for relative scale. This can be demonstrated by a very simple set-theory argument (Oldershaw, 1981*b*): only under such circumstances could the levels of substructure for analogues be matched in a one-to-one manner. In the case of an unbounded cosmological hierarchy such as that proposed by the SSCM, the universe cannot have a "center," nor can it have an absolute reference frame, nor can it have a "beginning" or an "end." Furthermore, the concept of cosmological "homogeneity" must be superseded by the concept of approximate "homogeneity" applicable to particular well-defined finite regions of space-time-scale. The same limitations also apply to the concept of cosmological "isotropy."

## 2.5. The Enigmatic Dark Matter

Since Paper I only considered classes of systems whose basic properties have been reasonably well characterized, a discussion of the well-known dark matter problem was temporarily deferred; this very important issue will now be examined within the context of the SSCM. Over the past 10 years astronomers have repeatedly demonstrated that the rotational motions of galaxies appear to extend well beyond the luminous regions of those galaxies (Faber and Gallagher, 1979), that at least 50% of the matter in the disk of our galaxy is in an unknown "dark" form (Bahcall, 1984), and that various dynamical observations on the galactic scale are consistent with the hypothesis that galaxies have vast haloes of matter that cannot be directly observed (Trimble, 1988). These new and totally unexpected findings seem to suggest that at least 90% of the mass in galactic systems, and therefore $\geq$90% of the matter in the observable universe, is in an unknown form that emits very little light as compared to previously identified forms of matter. Astronomers and physicists have proposed a large number of hypothetical candidates for the form of the dark matter (Oldershaw, 1986*c*), but there are very few theories that definitively predict what the dark matter *must* be. The SSCM unambiguously predicts (Oldershaw, 1986*b,c*) that the dark matter is primarily composed of ultracompact objects that all have roughly the same mass of approximately $0.145 M_\odot$. These ultracompact objects are stellar scale analogues of the proton, and since the latter objects constitute $\geq$90% of a representative sampling of the atomic scale, the former objects are likewise expected to constitute $\geq$90% of a cosmologically representative sampling of the stellar scale. Because those objects are in an ultracompact state, i.e., black holes, they emit very little light, although black holes can emit low levels of blackbody radiation (Misner *et al.*, 1973) and x-rays resulting from accretion of matter onto the object (Oldershaw, 1986*c*). Aside from the key prediction that these objects have a mass of about $0.145 M_\odot$, it has also been predicted that they have radii of about $0.4 \times 10^5$ cm and typical x-ray luminosities (due to the accretion of interstellar matter) of about $10^{29}$ or $10^{26}$ erg/sec, depending on whether they reside in the galactic disk or halo, respectively (Oldershaw, 1986*c*). While the stellar scale proton analogues are predicted to account for about 90% of the dark matter objects, stellar scale $He^{2+}$ analogues with masses of about $0.58 M_\odot$ should account for another 9% of the dark matter objects and the remaining 1% should be in the form of stellar scale analogues to more massive atomic nuclei. There are two other predicted but undiscovered classes of stellar scale objects that should also be present in significant numbers. One is the class of stellar scale analogues to the electron. In terms of abundance their number per galaxy should be slightly larger than the number of stellar scale $H^+$ and $He^{2+}$ analogues per galaxy, but since they are more than three orders of magnitude less massive, they make a relatively small contribution to the total mass of the dark matter. The masses and Schwarzschild radii of these low-mass black holes are predicted to be approximately $7 \times 10^{-5} M_\odot$ and about 20 cm, respectively. Analogues to hydrogen atoms in states characterized by very high values of $n$ and $l$ constitute another class of predicted stellar scale objects that contribute to the dark matter. These objects would consist of a stellar scale proton analogue orbited by an electron analogue that is distributed in a planetary system like that of the Solar System. Radii for the planetary distributions of these systems are expected to range from about $10^{12}$ to as large as $10^{14}$ cm, and extensive magnetic fields, reminiscent of those found in the Solar System, are to be expected. These analogues to highly excited H atoms would be difficult to detect since their nuclei are black holes and their planetary systems have very low luminosities. Their total number in a typical galaxy is estimated to be of the same order of magnitude as the number of visible stars, if the available stellar scale sample analogously reflects the relative cosmological abundances of protons, highly excited H, and H in moderate-to-low energy states.

### 2.6. Metagalactic Phenomena

According to the SSCM, the region of space-time that astronomers conventionally refer to as the "universe" is actually an incredibly small region within a single metagalactic scale object, the latter being one of a countably infinite number of objects comprising the cosmological scale that is immediately "above" the galactic scale. To get a proper understanding of just how relatively infinitesimal the observable region of this metagalactic object is, consider an analogy between this metagalactic object and a massive stellar scale star with a radius of roughly $10^{13}$ cm (as will seen below, this is not an arbitrary choice of analogies). Then the radius of the metagalactic object would be on the order of $10^{48}$ cm, while the radius of the observable portion of this object is roughly $10^{28}$ cm. Therefore, the radius of the observable volume is $10^{-20}$ times the radius of the object itself, and if the metagalactic object is analogously viewed as a large star, then the observable portion of that "star" has a radius that is not much larger than that of a single atom. Throughout this discussion of galactic and metagalactic objects the conceptual device of treating the metagalactic object as a "star" and its galactic scale components as "atomic scale objects" will be employed. This device allows one to interpret phenomena taking place on an unimaginably large scale in terms of more familiar phenomena, and it is highly in keeping with the SSCM's relativity of cosmological scale which allows one to view a galaxy with equal validity as an atomic nucleus, a neutron star, or a galaxy. In a previous paper (Oldershaw, 1986*c*) it was proposed that one could obtain rough estimates of the local galactic scale temperature, density, and composition within the observable region of the metagalactic object. The average "peculiar" velocity for observable galaxies can be used to infer a galactic scale temperature in the range of $10^7$–$10^9$ K, depending on uncertainties in estimates of the average mass of a galaxy and the average galactic scale "peculiar" velocity. The local galactic scale density was estimated to be in the range of $10^{10}$–$10^{11}$ g/cm$^3$, depending on uncertainties in the estimated average galactic mass and the estimated total number of galaxies within the observable region. Based on the observed distribution of galactic radii, it can be inferred that, in contrast to the usual atomic scale abundances, the local galactic scale composition includes a substantial fraction of galactic scale analogues to nuclei that are more massive than helium. This combination of estimated galactic scale parameters, combined with the strong evidence for high-velocity global expansion within the observable region, serves to greatly reduce the number of potential choices for an appropriate stellar scale analogue to the metagalactic system. It was suggested (Oldershaw, 1986*c*) that one possible stellar scale analogue wherein one can find comparable atomic scale temperature, density, composition, and expansion velocity values is the interior of a massive star undergoing a Type II supernova. If the expansion of the observable region of the metagalactic system did begin about 20 billion years ago, as is currently thought, then the required scaling by a factor of $\Lambda^{-2}$ yields an estimated *stellar scale* value of about $10^{-18}$ sec for the amount of time that has passed since the observable region of the metagalactic "star" began its expansion. The interpretation of the expanding "universe" in terms of a metagalactic scale supernova should be regarded as a tentative speculation, however, since the derivations of the galactic scale parameters involve theoretical assumptions that could be wrong, there are significant empirical uncertainties in the derivations of the galactic scale parameters, the current theoretical modeling of the physical phenomena occurring deep within a supernova just after the explosion may not be entirely satisfactory, and even if the estimated galactic scale parameters are reasonably accurate, the proposed interpretation might not be a unique one.

### 2.7. Galactic Structure and the Cosmos As a Discrete Self-Similar Hierarchy of Black Holes

In Paper I it was mentioned that at first inspection the cosmological hierarchy appears to be highly asymmetric, since the ratio of the mass of a neutron star to the mass of its atomic scale analogue (a nucleus) is of the order of $10^{56}$, but the ratio of the conventional mass of a galaxy (the neutron star's galactic scale analogue) to the mass of a neutron star is of the order of $10^{12}$. The SSCM asserts (Oldershaw, 1986*b*,*c*) that this apparent asymmetry is an artifact caused by the incorrect use of the stellar scale gravitational coupling constant $G_0$ in determinations of galactic masses instead of a proper galactic scale $G_1$ ($\sim 10^{-39} G_0$). When a properly scaled $G_1$ is substituted for $G_0$ in conventional galactic mass calculations, then the estimated galactic masses are closer to the enormous values ($10^{88}$-$10^{90}$ g) predicted by the SSCM, and the hierarchical asymmetry is nearly removed (see Section 4). If $G_0$ is the appropriate coupling constant for galactic scale systems, then the galactic masses predicted by the SSCM are laughably overestimated. But if $G_1$ is the correct coupling constant as proposed by the SSCM, and currently no empirical method has been devised for testing which coupling constant is correct, then it is our conventional estimates of galactic masses that are wildly inaccurate.

If galaxies actually contain as much mass as $10^{88}$-$10^{90}$ g, then there is only one place that this incredibly large amount of "extra" mass could be located such that its presence would not already have been very obvious: a singularity at the center of each galaxy. Since the SSCM proposes that galaxies are black holes, a central singularity is to be expected. Astronomers think that collapsed objects are present at the centers of galaxies, but their physical properties are not well known at this time. According to the SSCM, the structure of a galaxy is basically as follows. Virtually all of the galactic mass is in the form of a singularity at the galactic center. The bulge, disk, and halo of stars represent an infinitesimally fine "mist" of stellar scale objects within the galaxy's Schwarzschild radius. This "mist" of stellar scale objects would account for only about $(10^{45}\text{ g}/10^{89}\text{ g})\,(100) = 10^{-42}\%$ of the total galactic mass, a truly infinitesimal percentage.

Given this inferred galactic structure and the hypothesis that fully ionized systems account for at least 90% of all systems on all scales, as is known to be the case on the atomic scale, it can be concluded that the cosmological hierarchy is dominated by a nested self-similar hierarchy of discrete black holes. Consider a first approximation SSCM that *only* includes fully ionized matter on all scales. Galaxies have the structure just described, stellar scale objects within each galaxy (approximately $10^{13}$ objects accounting for $10^{-42}\%$ of the mass) are black holes with structures that are exactly self-similar to their galactic scale analogues, the infinitesimally fine "mist" of atomic particles within the stellar scale black holes are likewise self-similar analogues, and so on without limit on higher and lower scales.

It is immediately clear that the composite/component type of hierarchy, which characterized versions of the SSCM before the 1985 scaling laws were derived, is not a valid description of the hierarchical organization of the cosmos. In a composite/component hierarchy a scale-$N$ system is entirely composed of scale-$(N-1)$ systems that retain their individual identities, and the scale-$N$ system is one of the components that entirely make up a scale-$(N+1)$ system. But this is certainly not the case for post-1985 models, since all but $10^{-42}\%$ of the mass of a scale-$N$ system is in the form of an *elemental* central singularity that by definition cannot have any scale-$(N-1)$ structure.

Systems that are not fully ionized apparently account for $\leq 10\%$ of the systems on any scale. At present the SSCM cannot uniquely specify the internal structure of such systems; instead, this model currently offers two general alternatives for their structures. This uncertainty stems from the possibility that when fully ionized particles (black holes) become bound into atomlike systems, the original black hole structures *may* metamorphose into objects that are no longer fully collapsed, but rather have extensive mass distributions. For example, the stellar scale electron is hypothesized to be a black hole when it is unbound, but in stellar scale atoms the electronic mass can apparently be widely distributed in spherical shells, planetary systems, etc. It is conceivable, therefore, that the compactness of nuclear objects in atomlike systems has decreased to the point that their radii exceed their Schwarzschild radii, and that they become composites of lower scale systems (with the conventional model of a neutron star representing a reasonable approximation to this structure). If the hypothesized "metamorphosis" strikes one as being highly implausible, then one should remember that a metamorphosis of the "universe" from a fully collapsed state to a highly extended state is the *sine qua non* of the nearly universally accepted big bang paradigm. One possible structure for a scale-$N$ system that is not fully ionized consists of an ultracompact nucleus (scale-$N$ black hole) that is surrounded by a distributed electronic system that is primarily composed of scale-$(N-1)$ black holes. The other possible structure for a scale-$N$ system that is not fully ionized involves an extended ($R > R_{\mathrm{Sch}}$) nuclear object and an extended electronic system that are *both* primarily composed of scale-$(N-1)$ black holes. This uncertainty over the structure of the cosmologically "rare," but nevertheless important, incompletely ionized systems is an unresolved issue that will be discussed again in Section 4.

From the preceding discussion of structure in both bound and unbound systems it can be concluded that the SSCM views any system on any scale as a hierarchical collection of black holes, though a scale-$N$ system need not be, and perhaps need not contain, a scale-$N$ black hole. The different states of systems are distinguished according to the degree to which mass is distributed among singularities on different scales of the system's internal structure.

## 3. DEFINITIVE PREDICTIONS OF THE SSCM

### 3.1. The Dark Matter

Given the fact that $\geq 90\%$ of the mass on the atomic scale is in the form of "bare" atomic nuclei, the most straightforward and definitive prediction (Oldershaw, 1986*b*,*c*, 1987) of the SSCM is that the conventional galactic dark matter (not including singularities in galactic nuclei) is primarily composed of ultracompact ($R \approx R_{\text{Sch}}$) stellar scale objects that are analogues to atomic scale nuclei. Since numerical abundances for the atomic scale nuclei are about 90% protons, about 9% helium nuclei, and about 1% heavier nuclei, it can be predicted that about 90% of the dark matter objects have masses tightly clustered around a value of $0.145 M_{\odot}$, that about 9% of the dark matter objects have masses of about $0.580 M_{\odot}$, and that about 1% of the dark matter objects have masses between $0.73 M_{\odot}$ and $43.5 M_{\odot}$ (in near multiples of $0.145 M_{\odot}$). A related prediction is that there is a discrete mass spectrum for stars, in exact analogy to that for atomic masses. However, uncertainties in stellar mass estimates may prevent tests of this prediction for a considerable time. An unexplained cutoff (Waldrop, 1987) in the distribution of stellar masses below the lower limit of the $M$ dwarf mass range is consistent with SSCM expectations. Stellar scale electron analogues should be very numerous (even more numerous than nuclei analogues), but at a mass of about $7 \times 10^{-5} M_{\odot}$ the contribution to the total mass of the dark matter is very small compared to the contributions of proton and alpha-particle analogues. The predicted accretion-generated x-ray luminosities for stellar scale proton analogues in the galactic disk and halo are about $10^{29}$ and about $10^{26}$ erg/sec, respectively (Oldershaw, 1986*c*). Predicted x-ray luminosities for $\text{He}^{2+}$ analogues in galactic disks and haloes are about $10^{30}$ and about $10^{27}$ erg/sec, respectively (Oldershaw, 1986*c*). Given that the spin angular momentum of a proton is $h/4\pi$, where $h$ is Planck's constant, one can calculate a classical rotation period that is roughly on the order of $10^{-23}$ sec, and this implies that the rotational velocity at the "surface" of the proton approaches the velocity of light. The SSCM proposes that the stellar scale proton analogue should have an equally high rotational velocity, which gives a predicted spin period as short as $10^{-5}$ sec for a radius of $0.4 \times 10^{5}$ cm. However, it must be borne in mind that the classical rotation assumptions involved in this calculation may introduce significant errors when applied to rapidly rotating black holes. The range of radii for stellar scale nucleus analogues is about $0.4 \times 10^{5}$ to $4 \times 10^{5}$ cm, and the number of these objects in a typical galaxy should be roughly 10–100 times the number of luminous stars.

Detecting the predicted dark matter candidates will be difficult, but these ultracompact objects should reveal themselves through gravitational microlensing and the emission of high-energy radiation: x-rays, gamma rays, and possibly cosmic ray particles. There are several gravitational lensing experiments that can potentially test the SSCM's dark matter predictions (Oldershaw, 1989*d*). It is expected that the predicted stellar scale black holes in our galaxy would cause observable gravitational amplification of the luminosities of stars in nearby galaxies at a rate of about 9 events per year if $10^6$ stars were monitored (Oldershaw, 1987). Unexplained "companions" that subsequently disappear [i.e., VB8b (Schorn, 1987) and the "companion" to SN 1987a (Phinney, 1988)] could represent serendipitous observations of such events. Microlensing of very distant sources, such as quasars, BL Lac objects, and active galaxy nuclei by stellar scale black holes in the haloes of intervening galaxies can be expected to occur. In fact, it has been suggested that a number of the peculiar properties of these very distant sources, such as their variability, high luminosity, anomalous association with galaxies at discrepant redshifts, etc., might be due to microlensing by dark matter objects (Ostriker and Vietri, 1985; Oldershaw, 1989*d*). Very recently a group of astronomers has identified an extremely rare example of a distant quasar being lensed by a relatively nearby and almost perfectly aligned galaxy (Schneider, *et al.*, 1988). The group comments that this rather improbable galaxy/quasar alignment may represent the optimum circumstances for investigating microlensing; sensitive monitoring of the different quasar images is expected to yield the first estimates of the absolute masses of dark matter objects. It has also been predicted that stellar scale black holes could cause observable effects in the light curves and polarizations of supernovae (Schneider and Wagoner, 1987). It is hoped that one or more of these methods will eventually yield accurate data on the physical properties of the dark matter constituents.

A more direct method of observing the predicted stellar scale black holes will be possible when very sensitive x-ray detectors with high resolution [such as the proposed AXAF satellite (Giacconi, 1987)] are put into operation. The SSCM predicts that a new and very populous class of x-ray sources, with the luminosities predicted above, will pervade our galaxy. An excess ridge of x-rays from the inner region of the galactic disk is suspected of being due to unresolved low-luminosity "point" sources (Warwick *et al.*, 1985). It is also known that $\geq 50\%$ of the observed x-ray background is unaccounted for, and a new class of low-luminosity discrete sources is the most viable explanation (Giacconi, 1987). A vast population of stellar scale black holes in the galactic halo may generate this "background" radiation. Enigmatic gamma-ray bursts, for which sources have not been identified (McBreen and Metcalfe, 1988), may be associated with the predicted black holes. And of course, the true nature of the dark matter objects may be revealed by methods other than the ones discussed above, such as the observed multiple imaging of pulsars (Wolszczan and Cordes, 1987) or the "occulting" of galactic radio sources (Fiedler *et al.*, 1987).

One important caveat to this crucial suite of SSCM predictions must be mentioned before turning to a new subject. The dark matter predictions rely on the assumption that the observable stellar scale objects will be predominantly in fully ionized states, as is the case for atomic scale objects. Within the context of the SSCM this assumption is certainly valid for a cosmologically representative sample of stellar scale objects, but it may not hold for the limited sample available to observation. The physical conditions within galaxies might be such that the majority of the observable stellar scale objects are highly excited, but not fully ionized. If this were the case, and the prevalence of multiple star systems seems to suggest that it might be, then the predicted mass spectrum for the dark matter remains virtually the same, but the objects (highly excited stellar scale H and He analogues) would have bound electronic systems, and thus more extended and complicated mass distributions and fields. However, it is presently considered far more likely that the assumption of complete ionization for the majority of stellar scale objects is correct.

### 3.2. Radius of the Electron

Within the context of the SSCM, unbound subatomic particles are regarded as ultracompact objects with radii approximately equal to their properly scaled Schwarzschild radii (Oldershaw, 1987). It has been shown that the scaled Schwarzschild relation yields a good approximation to the charge radius of the proton (Oldershaw, 1986*b*), and SSCM predictions are also of the right order of magnitude for pi and $K$ mesons. It has been predicted (Oldershaw, 1987) that the *unbound* electron has a radius of approximately $4\times10^{-17}$ cm, not far below the current resolution limits. Verification of this prediction would have profound implications. It is possible that electrons and their analogues on other scales are "naked singularities" which do not have event horizons or conventional radii, but most physicists regard naked singularities as unphysical mathematical solutions of general relativity. It should also be mentioned that the scaled Schwarzschild equation applies to uncharged and nonrotating sources, and therefore relativistic corrections can be expected to alter the radius prediction given above. However, it seems reasonable to assume that $10^{-17}$ cm is the correct order of magnitude for the radius of the unbound electron.

### 3.3. Stellar Morphologies

The SSCM proposes a rigorous analogy between atoms and stars. The Schrödinger $|\Psi|^2$ function is interpreted as representing the distribution of the charged $N=-2$ systems that compose the extended electronic structure of bound atomic systems. A stellar scale analogue to the $|\Psi|^2$ function would represent the distribution of charged atomic scale $(N=-1)$ systems that make up the noncompact electronic structure of a star. Therefore, the striking morphologies of the atomic scale $|\Psi|^2$ functions (White, 1931) should have exact analogues in the morphologies of stellar "atmospheres." To date it has not been possible to observe the detailed morphologies of stars because of their great distances, but in the foreseeable future it will be possible to resolve the outer structures of stars in our region of the galaxy. Since no other theory predicts such radical stellar morphologies, verification of this prediction would constitute extremely powerful support for the SSCM. It has been noted previously that the structures being ejected from stars in planetary nebulae [interpreted within the SSCM as stellar scale ionization (Oldershaw, 1986b)] have unique and unexplained morphologies that resemble the basic morphologies of atomic $|\Psi|^2$ functions (Oldershaw, 1982. 1986*b*).

### 3.4. A New *R* Versus *n* Relation for Rydberg Atoms

In Paper I the proposed analogy between variable stars and Rydberg atoms undergoing energy state transitions was discussed, and two period-radius relations that hold for Rydberg atoms were shown to give good approximations to the general period-radius relations of variable stars when the parameters in the equations are scaled according to the SSCM rules. Several classes of variable stars obey a third period-radius relation that is related to but different from the first two period-radius relations. As a result, it can be unambiguously predicted (Oldershaw, 1988, 1989*b*) that subsets of Rydberg atoms have radii that are approximated by $R \approx 4n^2a_0$, in addition to the already known relations $R \approx 2n^2a_0$ had $R \approx n^2a_0$.

### 3.5. Cores of "Coreless" Planetary Nebulae and Supernovae

A previous paper (Oldershaw, 1986*b*) presented the argument that the stellar evolutionary sequence red giant star-planetary nebula-white dwarf remnant is analogous to an ionization event in a highly excited multielectron atom that leaves the nearly completely ionized ion in close to its lowest energy state. The author noted that similar stellar scale ionization events that leave only bare nucleus analogues should occur and could be identified as planetary nebulae that *appear* to be "coreless." Indeed, a significant fraction of planetary nebulae do appear to be "coreless" (Pottasch, 1984) and the SSCM unambiguously predicts that stellar scale nucleus analogues (black holes) will be found at the centers of many of these systems. The most promising method for detecting such objects would be to search the centers of "coreless" planetary nebulae with very sensitive high-resolution x-ray detectors.

Because of their enormous energy output and mass ejection, the SSCM proposes that supernovae are stellar scale analogues of highly excited atoms undergoing radioactive decay events. A large percentage of supernovae *appear* to leave no core object (Trimble, 1985), such as a neutron star. The SSCM again predicts that many "coreless" supernovae remnants will be found to have very massive black holes at their centers and that these objects should be detectable as low-luminosity x-ray "point" sources.

### 3.6. A 52-sec Preferred Period for White Dwarf Star Oscillations

In a previous paper (Oldershaw, 1989*c*) it was noted that the period distribution for variable white dwarf stars (the majority of which are interpreted as $He^+$ analogues) suggests that there are preferred oscillation periods at roughly $250 \pm 100$ and $850 \pm 100$ sec. As expected from SSCM considerations, these preferred periods and two "persistent" oscillation periods for $He^+$ ions are related by the scaling factor $\Lambda$. A third "persistent" oscillation period for $He^+$ has not been matched by a corresponding preferred period for white dwarfs, and therefore it has been predicted that larger samples of periods for variable white dwarf stars will eventually manifest a preferred period at about 52 sec.

### 3.7. Very Large Spin "Glitches" for Pulsars

Pulsars have been identified as analogues to atomic scale nuclei in excited states and it has been shown that the magnetic dipole moments of neutron stars and atomic nuclei are related by the SSCM scaling relations (Oldershaw, 1987). Both pulsars and exited atomic nuclei exhibit a remarkable phenomenon called "glitches" wherein the steadily decreasing rotation rate of the object suddenly jumps to a higher value and then returns to steady decrease from the higher spin rate (Stephens, 1985). A quantitative measure of a "glitch" is given by $\Delta P/P$, where $P$ is the rotation period and $\Delta P$ is the magnitude of the sudden jump. In excited atomic nuclei $\Delta P/P$ can be as high as 0.1, whereas the largest "glitch" observed (Lyne, 1987) for a pulsar was on the order of $10^{-6}$. Therefore, the SSCM unambiguously predicts that pulsars can exhibit "glitches" with $\Delta P/P$ values up to 0.1, and no other theory makes such a prediction. A caveat here is that very large "glitches" have only been observed in very massive atomic nuclei, and stellar scale nucleus analogues of correspondingly high mass should be extremely rare. Moreover, even among a population of very high mass pulsars the probability of a "glitch" event with $\Delta P/P = 0.1$ may be very small. Nevertheless, one can expect that as more pulsars are discovered and monitored the observed $\Delta P/P$ maximum will keep increasing, and perhaps a very large "glitch" will be observed in spite of the odds.

### 3.8. Derivation of Solar System Parameters Using Scaled Quantum Mechanics

As mentioned in Section 2.2, the SSCM regards the Solar System as a stellar scale analogue to a neutral, but highly excited, Li atom characterized by the quantum numbers $n_3 \approx 168$, $l_3 \approx n_3$; $n_2 = 5$, $0 \leq l_2 \leq 2$; $n_1 \approx 1$, $l_1 \approx 0$. This analogy is unambiguously derived from the SSCM scaling equations, the Sun's physical properties (mass, radius, shape), the radius of Jupiter's orbit, and the planarity of the planetary system (Oldershaw, 1986*b*). It can be predicted that if a quantum mechanical wave function is calculated for a Li atom in the state approximated above, and if the physical parameters are scaled according to the SSCM rules, then there should be a quantitative correspondence between the physical properties described by the wave function (reinterpreting $|\Psi|^2$ as the distribution of the electron's lower scale constituents) and those observed for the Solar System. At a minimum, one would expect to be able to derive the orbital radii and masses of the planets, and to identify the $22.27 \pm 0.08$ year solar magnetic cycle (Dicke, 1978) with a rigorously analogous atomic scale phenomenon. To the extent that one could accurately infer the values of all four quantum numbers for each of the three electron structures, quantitative agreement between the scaled quantum mechanical calculations and the more detailed properties of the solar system (i.e., planetary spin periods, orbital inclinations, properties of moon systems, etc.) should become possible.

### 3.9. Generality of the Scale Transformation Equations

The self-similar scaling equations of the SSCM provide the potential for a virtually limitless number of predictions. If one can unambiguously identify analogous objects or phenomena on different cosmological scales and can *accurately* (precision is highly desirable, but accuracy is far more important in this context) measure corresponding parameters on both scales, then it can be predicted that these measurements will be correlated in the manner specified by the SSCM scaling rules.

## 4. UNRESOLVED ISSUES AND PROBLEMS

As is the case with most new theories, the SSCM offers solutions to some previous problems, but also raises a number of new problems. In this review the most important and fundamental problems associated with the SSCM will be addressed; until these major concerns are adequately resolved, one can postpone work on less compelling problems.

### 4.1. Galactic Masses

As mentioned in Section 2.7, the SSCM proposes (Oldershaw, 1986*c*) that a typical spiral galaxy has a mass on the order of $10^{89}$g, while conventional analyses of galactic masses yield a typical value on the order of $10^{45}$g. Thirty-nine orders of magnitude of this truly spectacular disparity between the SSCM and conventional values would be accounted for if the stellar scale gravitational "constant" $G_0$ must be replaced by a properly scaled $G_1$ in galactic mass determinations. Conventional estimates of galactic masses involve dynamic models wherein $G_0$ enters as a constant of proportionality. The SSCM asserts that a scaled $G_1$ ($\approx 10^{-39} G_0$) must be used in these dynamic models, and if this is done, then the conventional estimate is revised upward to a value of roughly $10^{84}$g, which is much closer to the SSCM prediction. However, there is still a factor of $10^5$ disparity between the SSCM prediction and the SSCM-revised conventional estimate. At present this discrepancy cannot be explained, although a likely possibility is that one (or more) of the dynamical assumptions that underlie the conventional estimate (Hodge, 1966) is inappropriate. Within the context of the SSCM, this can be readily anticipated, since the conventional dynamical models consider only simple weak-field gravitational interactions. The SSCM views galactic interiors as regions wherein very strong galactic scale gravitational fields exist, and the SSCM views galactic interactions as involving equally strong galactic scale electromagnetic fields. If a discrepancy of $10^5$ seems excessive, consider that the standard model of particle physics predicts a cosmological constant that is $10^{46}$ times larger than the observationally based upper limit for this parameter (Abbott, 1988). It remains to be seen how well the SSCM viewpoint holds up when the physics of the SSCM is more fully developed and applied to the case of galactic mass estimates.

### 4.2. Scale Transitions

There is an unresolved problem concerning the nature of the transitions between cosmological scales, and this problem can be illustrated by considering how the gravitational "constant" $G_N$ varies with cosmological scale. Taking the stellar scale as the reference scale and using normal cgs units, the SSCM suggests that $G_0$ (the usual Newtonian gravitational constant) applies within stellar scale systems, $G_{-1}$ ($\approx \Lambda^{2.174} G_0$) applies within atomic scale systems, and $G_1$ ($\approx \Lambda^{-2.174} G_0$) applies within galactic scale systems. It has been shown that in determining the radius of the proton via the scaled Schwarzschild equation one must use $G_{-1}$. Since the interactions of the major components of the Solar System involve $G_0$, the SSCM requires that $G_{-1}$ is also the correct gravitational constant within atoms. However, laboratory experiments that measure the gravitational constant using interactions between relatively small spheres ($\sim$100 kg) of matter show that $G_0$ applies to these interactions. Therefore, it would appear that the SSCM inevitably leads to the conclusion that *inside* an atomic scale system $G_{-1}$ is appropriate, but that interactions *between* separate atomic scale systems, or collections of atomic scale systems, involve $G_0$. A tentative general rule is as follows. To determine the $N$ value for $G_N$ (or other "constant"), determine the smallest physical system of which the relevant test region is a part, and the $N$ value for that system is also the correction $N$ value for $G_N$. But this scaling rule leads to the disconcerting conclusion that gravitational fields have finite ranges or can be "neutralized" as is the case with electromagnetic phenomena. It is also difficult to imagine exactly where and how the transitions between $G_N$ and $G_{N+1}$ take place, since relevant systems such as atoms, stars, and galaxies do not appear to have well-defined boundaries. The above scaling rule also leads to the conclusion that *separate* stellar scale systems interact via $G_1$ rather than $G_0$, which is a radical departure from current assumptions. Consider two stars initially separated by several light years but moving toward eventual coalescence into a close binary system. If the above-mentioned general rule is correct, then initially their interaction is characterized by $G_1$, but after coalescence they interact with a coupling constant of $G_0$. Where and how does $G_N$ increase by about 38 orders of magnitude? Clearly, if the SSCM is a valid approximation of how nature works, then some very revolutionary changes to currently accepted physics will be required. At present, solutions to the problems of scale transitions are not yet on the horizon.

### 4.3. Galactic Scale Electrons

The SSCM views galactic scale phenomena as analogous to a dense hot plasma of fully ionized particles (Oldershaw, 1986*c*), and galaxies are interpreted as analogues to bare atomic nuclei. The galactic scale analogues to free electrons, which should be numerous constituents of the galactic scale plasma, have not been identified in previous papers. Here it is proposed that globular clusters may represent galactic scale analogues to free electrons. The SSCM predicts that the atomic scale electron has a radius of approximately $4 \times 10^{-17}$ cm and if this is correct, then the galactic scale electron analogues should have radii of about $\Lambda^2$ times the atomic scale value, or approximately 4 parsec ($\approx 10^{19}$ cm). Radii for globular clusters are not well defined: their tidal radii vary from 20 to 200 parsec, and their *estimated* core radii (radius at which the surface brightness levels off) vary from 1 to 15 parsec. If the core radii are the more appropriate radii for the electron analogy, as seems reasonable, then the average core radius is within about a factor of 2 of the predicted radius for the galactic-scale free-electron analogue. Globular clusters are very numerous galactic scale constituents and they cluster around galaxies (analogues to positive ions). However, a more detailed investigation of the proposed analogy needs to be carried out. Large galaxies often have retinues of hundreds of globular clusters. Is this consistent with the number of electrons that could be expected to cluster around positive ions in a hot dense plasma? Are other quantitative parameters of globular clusters, in addition to their radii, consistent with their being self-similar electron analogues?

## 4.4. Stellar Scale Structure and Evolution

If the SSCM correctly interprets stars (and their planetary systems) as analogues to atoms, then one is confronted with the problem of how the stellar scale electron can vary among ultracompact states (for the unbound case), diffuse states (plasma shells around stellar nuclei), and planetary states (as seen in the Solar System), depending on its interactions with other stellar scale objects. In quantum mechanics the function $|\Psi|^2$, interpreted as the spatial distribution of the probability for finding the electron, does vary in a related manner, but the SSCM requires that $|\Psi|^2$ be reinterpreted as the spatial distribution of the mass/charge associated with the electron or its scale $N = -2$ constituents. Currently it is thought that the collapse of matter into an ultracompact state (black hole) is virtually irreversible, and therefore the SSCM implication that the stellar scale electron analogue may reversibly change from ultracompact to extended states appears to conflict with the current theoretical models of black hole physics. Part of the conflict may be due to the fact that conventional wisdom largely dismisses a major role for stellar scale electromagnetic phenomena, whereas the SSCM predicts that electromagnetism and gravitation are of comparable "strengths" and have comparable dynamic influences on each cosmological scale.

A related problem is the internal structure of stars. The SSCM identifies stars and atoms as rigorously self-similar analogues and since atoms appear to have $\geq 99.9\%$ of their mass within a highly compact nucleus, the SSCM appears to require the very radical hypothesis that the distribution of mass in stars is similarly organized. Such a suggestion is tantamount to astrophysical heresy. The other option, that atomic nuclei are not ultracompact, seems to be contradicted by very strong empirical support for ultracompact nuclei in atoms. It seems, therefore, that the hypothesis of black holes (or at least very compact objects like neutron stars) existing at the centers of all stars and accounting for $\geq 99.9\%$ of their masses is difficult to avoid if exact self-similarity is retained in the SSCM. While such a hypothesis strains credibility to the limit, it should be remembered that a major predictive test of existing theoretical models of stellar interiors, the flux of solar neutrinos, has consistently yielded a worrisome factor-of-three discrepancy (Thomsen, 1988). Moreover, with regard to the *best known* star, the Sun, there are still many enigmas: the 22-year solar cycle with its remarkable magnetic field reversals, an apparently rapidly rotating solar core, a diverse array of oscillations with periods ranging from 5 to 160 min, corona and flare phenomena, etc. So perhaps there is more to learn about stellar interiors.

Finally, there is a major conflict between the current theories of stellar evolution and the predictions of the SSCM regarding stellar scale phenomena. This is such a wide-ranging problem that it will only be mentioned here along with several relevant comments; it is an important topic for future work on the SSCM. The current body of theoretical knowledge about stellar evolution is widely regarded as having strong empirical support and internal consistency. On the other hand, the processes involved in the "birth" of stars and the fate of stars at the "ends" of their "lives" are still matters of considerable conjecture and disagreement. The SSCM asserts that there is a complete and rigorous analogy between atomic scale phenomena and stellar scale phenomena. If this general analogy is valid, then the irreversible stellar "evolution" of current models must be radically reinterpreted so as to conform with the observed reversibility (or near reversibility) of atomic scale phenomena. For example, the key process of star "formation" would have to be nucleation by a preexisting stellar scale nucleus analogue. As is the case for atomic scale phenomena, creation and annihilation of stellar scale analogues to "elementary" particles would be possible, but stars would more typically represent systems undergoing atomlike changes of state rather than systems confined to irreversible evolutionary paths. In support of this radical reinterpretation of stellar scale phenomena one could cite the results of tests 1-8, 10-12, and 15-20 presented in Paper I, which are consistent with SSCM expectations. Analogies such as the one linking the red giant star-planetary nebula-white dwarf sequence to the sequence of events during ionization of an excited helium atom (Oldershaw, 1986*b*) and the strong analogy between variable stars and Rydberg atoms indicate that it may not be so naive to anticipate a comprehensive, rigorous correspondence between stellar and atomic phenomena. I can think of no *empirical* evidence that would rule out this general analogy, but on the other hand it is going to be a long and difficult task to achieve the radical reinterpretation of stellar phenomena that the SSCM requires.

### 4.5. Atomic Scale and Stellar Scale Dynamics

The dynamics of the macroscopic constituents of the Solar System are dominated by gravitational interactions. In a very highly excited Rydberg atom ($n \approx 168$, $l \approx n$), the SSCM's proposed analogue to the Solar System, the dynamics of the electronic structure are dominated by electromagnetic interactions, which appear to be qualitatively different from gravitational interactions. Yet the SSCM claims that these two analogous systems have equivalent dynamics (except for a huge scale change), and this is a major unresolved issue. There are strong analogies between the morphologies and the kinematics of the Solar System and a very highly excited Rydberg atom, and so it is not inconceivable that their dynamics are much more analogous than is currently believed. However, at present there is no adequate explanation of how this paradox is to be resolved within the context of the SSCM. A starting point for any attempt at a resolution would probably begin with the SSCM assertion that atomic scale gravitational interactions are on the order of $10^{38}$ times stronger than is currently believed.

### 4.6. Quantum "Weirdness" and the SSCM

Quantum mechanics introduced a number of decidedly counterintuitive ideas into physics, such as wave-particle duality, antiparticles, nonlocality, the uncertainty principle, and nonclassical spin. It is important to note that although quantum mechanics is a highly successful theory and the unusual quantum phenomena seem well documented, there is still no general agreement upon an atomic scale interpretation of quantum physics. Rather, there are at least five radically different interpretations of the quantum "weirdness" (Davies and Brown, 1986). To the extent that the quantum "weirdness" represents intrinsic attributes of atomic and subatomic phenomena, the SSCM requires that analogous physics can take place on the stellar and galactic scales. However, the SSCM has not yet offered definite insights into the resolution of the interpretive problems on the microscopic scale nor offered candidate examples of analogous phenomena on other scales. On the other hand, A. B. Datzeff has constructed a comprehensive reinterpretation of quantum mechanics based on the hypothesis that atomic scale particles and dynamics are the epiphenomena of a subquantum physics that involves similar particles and dynamics but on a vastly smaller scale (Jammer, 1974; Datzeff, 1984). Such a reinterpretation of quantum mechanics is very much in the spirit of the SSCM and demonstrates that there is a real possibility that the idea of cosmological self-similarity might lead to a major advance in our understanding of quantum "weirdness." Many of the counterintuitive atomic scale phenomena may be attributable to observing atomic scale systems from a different, higher cosmological scale. It is conceivable that *within* each cosmological scale the physical laws are classical, deterministic, and causal, and that quantum "weirdness" arises from the constraints and novelties of interscale observations. At any rate, the SSCM offers the hope that one general mechanics holds good for all scales of the cosmological hierarchy.

### 4.7. The Scaling Constant of the Mass Transformation Equation

As mentioned in Section 2.1, the constant $X$ in equation (3) has been equated with $\Lambda^D$ in past literature on the SSCM. This would certainly be appropriate if nature's hierarchy were of the composite/component type defined in Section 2.7. However, the SSCM seems to involve a hierarchy that is dominated by ultracompact objects on all cosmological scales, and therefore the arguments used to assume that $X = \Lambda^D$ may not be valid. On the other hand, the SSCM also seems to require that ultracompact objects on all scales can undergo reversible changes between fully collapsed states and states that do not involve full gravitational collapse. Therefore, the appropriateness of equating $X$ with $\Lambda^D$ is an unresolved issue.

### 4.8. Stellar Scale Photons

Atomic scale systems have discrete energy levels and when an atom makes a transition from a higher to a lower energy level a photon is emitted. Photons are treated as discrete packets of energy which can be absorbed by a second atom, thereby boosting that atom into a higher energy state. The SSCM proposes that rigorously analogous phenomena can take place on all cosmological scales, but the mechanism by which this occurs on the stellar scale has not been identified. The SSCM views variable stars as analogues to excited atoms undergoing energy level transitions, and variable stars commonly emit shells of matter. Other stars are known to eject highly collimated jets, and possibly discrete objects such as Herbig-Haro objects. However, a comprehensive model of stellar scale electromagnetic phenomena remains to be worked out.

### 4.9. Observability of Galaxies

Yet another unresolved issue for the SSCM is the simple fact that the interiors of galaxies and globular clusters are readily visible. The SSCM proposes that these systems are galactic scale black holes, and one would expect that their interiors would be within event horizons, and therefore unobservable. Possible resolutions of this problem could be based around the following ideas: that event horizons of scale-$N$ objects do not apply to scale-$(N-2)$ phenomena, that the galactic scale "particles" and their analogues on other scales are not fully collapsed, that the physical laws of black holes need amending, or that galactic scale "particles" and their analogues on other scales represent naked singularities which do not have event horizons and so have observable interiors (Newman and Joshi, 1988).

## 4.10. Mass of the Planetary System

One of the most worrisome empirical problems confronting the SSCM is the fact that the estimated mass of the Solar System's planetary system is roughly $2.7 \times 10^{30}$g, whereas according to the Solar System/Rydberg atom analogy the observed value should be approximately $7.8 \times 10^{-5} M_{\odot}$, or about $1.6 \times 10^{29}$g. Because this factor of 17 discrepancy appears in such a straightforward test of the SSCM, it is regarded as a serious challenge to the exact self-similarity version of the SSCM. No encouraging clues for a possible resolution have been identified. "Effective masses" of up to 100 times the electron mass have been proposed for electrons in some solid-state phenomena (Fisk *et al.*, 1986), but this "variation" in the electron's mass does not seem applicable to the problem at hand. A closely related discrepancy involving the angular momentum of the planetary system is as follows. Highly excited Rydberg atoms with $l \approx n$ have angular momenta approximated by $L_{-1} = \hbar[n(n-1)]^{1/2}$ (Rowe, 1987). In Section 2 the value of $n$ derived for the planetary system was 168, and the nearly circular and planar orbits are indicative of the $l \approx n$ case. Therefore, one may use $n = 168$ to derive an atomic scale $L_{-1}$ of approximately $1.8 \times 10^{-25}$ erg sec. Scaling this result by the SSCM transformations gives a predicted stellar scale $L_0$ of about $1.6 \times 10^{49}$ erg sec for the planetary system. However, the observed value of $L_0$ is approximately $3.1 \times 10^{50}$ erg sec, which is 19 times larger than the SSCM prediction.

## 4.11. Concluding Remarks on SSCM Problems

Even the author gets disheartened when reviewing the numerous unresolved problems facing the SSCM, but the successful results in the large number of retrodictive tests cited in Paper I and the SSCM's potential for unification provide compensatory encouragement. The definitive predictions given in Section 3 will eventually reveal whether or not the SSCM is the right cosmological path to follow. In science one must have much patience and a willingness to reserve judgement until available knowledge is sufficient to permit a well-informed judgement. A case in point is Lord Kelvin's mistaken "proof" that the Sun's age was on the order of $10^8$ years, thereby "falsifying" Darwinian evolution, which would take much longer. If the SSCM is valid, then the major unresolved problems listed in this section make it clear that the model could use the help of a very insightful mathematical physicist. At best, the SSCM is in a situation not unlike that of

Faraday's original electromagnetic field model, i.e., badly in need of a Maxwell, or better yet, an Einstein.

## 5. IMPLICATIONS OF COSMOLOGICAL SELF-SIMILARITY

### 5.1. A Global Transfinite Hierarchy

If the exact self-similarity version (strong principle of self-similarity) of the SSCM is valid, then nature is organized into a global hierarchy. This means that multiscaled organization is a fundamental and global property of nature rather than a secondary and local phenomenon (Oldershaw, 1985). It further means that the systems on any cosmological scale are as fundamental as their analogues on any other cosmological scale. The reductionist assumption of increasing "elementarity" with decreasing scale is valid *within* a cosmological scale or interscale region of the hierarchy, but it is definitely not valid for the hierarchy as a whole. Exact self-similarity also requires that the number of cosmological scales is infinite (Oldershaw, 1981*b*) and therefore nature is completely unbounded in terms of space, time, and scale. There are neither smallest nor largest systems, nor any beginning or end of the whole universe, but rather an infinite nesting of self-similar systems.

### 5.2. Absolute Scale Is Renounced

An inevitable consequence of exact cosmological self-similarity is that the concept of absolute scale can no longer be accepted; scale would be a purely relative property of natural systems. As discussed in Section 2.3, within the context of the SSCM dimensional units do not have absolute meaning. For example, there is an infinite number of *different* centimeters: one for each scale. In conventional physics if one wishes to specify the location of a point relative to a given origin in an *n*-dimensional space, then one must specify a quantitative value for each of the *n* dimensions. Within the context of the SSCM, however, this procedure specifies an infinite series of points, because the quantitative values of units no longer have absolute status. Therefore, in order to specify a unique point within the infinite set of points, one must also specify the cosmological scale of the units being used. If the strong principle of self-similarity is validated, then any theory that incorporates absolute spatial or temporal scales would have to be modified or replaced by a scale-invariant theory.

### 5.3. Equivalent Physics on All Cosmological Scales

The SSCM requires that the physical laws pertaining to analogous systems on all cosmological scales are rigorously equivalent. If one chooses to designate an arbitrary set of dimensional units as "absolute" units, then all physical laws must be covariant with respect to discrete changes in cosmological scale. If one chooses the more natural strategy of avoiding "absolute" units, then all physical laws must be invariant (Ohanian, 1976) with respect to discrete changes in cosmological scale.

### 5.4. All Is Geometry

A likely implication of the SSCM is that Einstein's vision of a unified field theory wherein all physical objects and their dynamics are attributable to the geometric properties of space-time will be vindicated. Physical objects would be modeled as black holes (purely geometric objects) or collections of black holes, and general relativity would represent a path-breaking demonstration that dynamics can be understood in a purely geometric context.

### 5.5. The Case of Approximate Cosmological Self-Similarity

If the self-similarity of analogue systems on different cosmological scales is approximate (but nontrivial), then a transfinite number of cosmological scales is no longer assured and the concept of absolute scale remains viable. Local examples of approximate and statistical self-similarity (Mandelbrot, 1982) are commonly observed in nature, but the ubiquity of self-similarity has yet to be explained from first principles. Likewise, the verification of approximate cosmological self-similarity would represent a major discovery about nature, but a largely enigmatic one. The predictions listed in Section 3 provide an initial battery of tests that will be helpful in deciding whether the special beauty and simplicity of exact cosmological self-similarity must be relinquished in favor of approximate cosmological self-similarity.

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